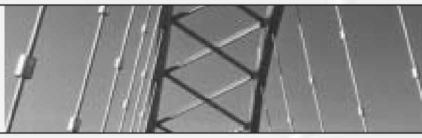


6

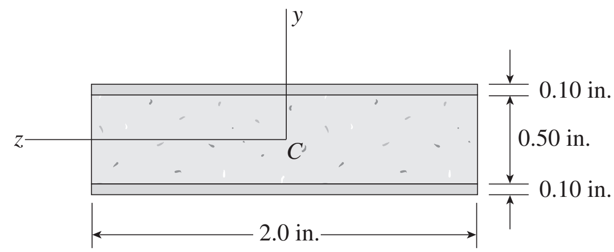
Stresses in Beams (Advanced Topics)

Composite Beams

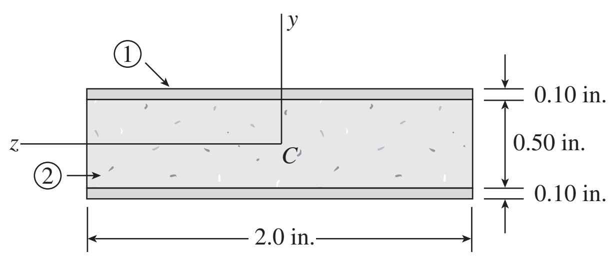
When solving the problems for Section 6.2, assume that the component parts of the beams are securely bonded by adhesives or connected by fasteners. Also, be sure to use the general theory for composite beams described in Sect. 6.2.

Problem 6.2-1 A composite beam consisting of fiberglass faces and a core of particle board has the cross section shown in the figure. The width of the beam is 2.0 in., the thickness of the faces is 0.10 in., and the thickness of the core is 0.50 in. The beam is subjected to a bending moment of 250 lb-in. acting about the z axis.

Find the maximum bending stresses σ_{face} and σ_{core} in the faces and the core, respectively, if their respective moduli of elasticity are 4×10^6 psi and 1.5×10^6 psi.



Solution 6.2-1 Composite beam



$b = 2$ in.

$h = 0.7$ in.

$h_c = 0.5$ in.

$M = 250$ lb-in.

$E_1 = 4 \times 10^6$ psi

$E_2 = 1.5 \times 10^6$ psi

$$I_1 = \frac{b}{12}(h^3 - h_c^3) = 0.03633 \text{ in.}^4$$

$$I_2 = \frac{bh_c^3}{12} = 0.02083 \text{ in.}^4$$

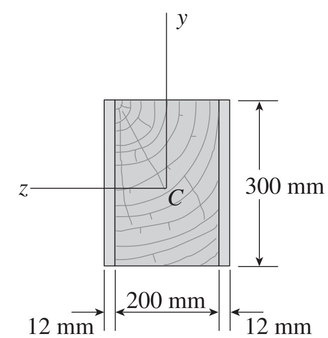
$$E_1I_1 + E_2I_2 = 176{,}600 \text{ lb-in.}^2$$

From Eq. (6-6a): $$\sigma_{face} = \pm\frac{M(h/2)E_1}{E_1I_1 + E_2I_2} = \pm 1980 \text{ psi} \quad \longleftarrow$$

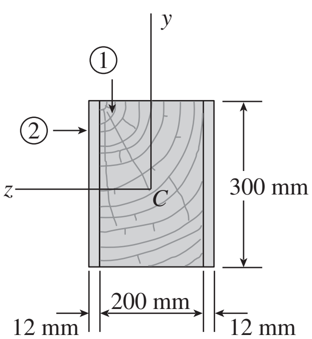
From Eq. (6-6b): $$\sigma_{core} = \pm\frac{M(h_c/2)E_2}{E_1I_1 + E_2I_2} = \pm 531 \text{ psi} \quad \longleftarrow$$

Problem 6.2-2 A wood beam with cross-sectional dimensions 200 mm × 300 mm is reinforced on its sides by steel plates 12 mm thick (see figure). The moduli of elasticity for the steel and wood are E_s = 204 GPa and E_w = 8.5 GPa, respectively. Also, the corresponding allowable stresses are σ_s = 130 MPa and σ_w = 8.0 MPa.

Calculate the maximum permissible bending moment M_{max} when the beam is bent about the z axis.



Solution 6.2-2 Composite beam



$b = 200$ mm
$t = 12$ mm
$h = 300$ mm
$E_1 = E_w = 8.5$ GPa
$E_2 = E_s = 204$ GPa

$(\sigma_1)_{allow} = \sigma_w = 8.0$ MPa
$(\sigma_2)_{allow} = \sigma_s = 130$ MPa

$$I_1 = \frac{bh^3}{12} = 450 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{2th^3}{12} = 54 \times 10^6 \text{ mm}^4$$

$$E_1I_1 + E_2I_2 = 14.84 \times 10^6 \text{ N} \cdot \text{m}^2$$

MAXIMUM MOMENT BASED UPON THE WOOD (1)

From Eq. (6-6a):

$$M_{max} = (\sigma_1)_{allow}\left[\frac{E_1I_1 + E_2I_2}{(h/2)E_1}\right] = 93.1 \text{ kN} \cdot \text{m}$$

MAXIMUM MOMENT BASED UPON THE STEEL (2)

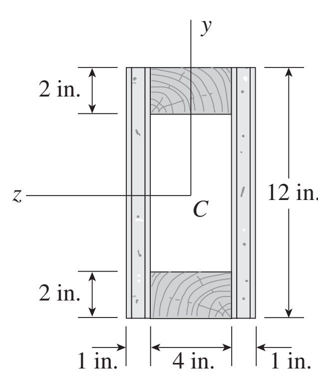
From Eq. (6-6b):

$$M_{max} = (\sigma_2)_{allow}\left[\frac{E_1I_1 + E_2I_2}{(h/2)E_2}\right] = 63.0 \text{ kN} \cdot \text{m}$$

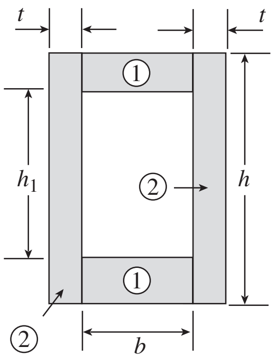
STEEL GOVERNS. $M_{max} = 63.0$ kN · m ←

Problem 6.2-3 A hollow box beam is constructed with webs of Douglas-fir plywood and flanges of pine as shown in the figure, which is a cross-sectional view. The plywood is 1 in. thick and 12 in. wide; the flanges are 2 in. × 4 in. (actual size). The modulus of elasticity for the plywood is 1,600,000 psi and for the pine is 1,200,000 psi.

If the allowable stresses are 2000 psi for the plywood and 1700 psi for the pine, find the allowable bending moment M_{max} when the beam is bent about the z axis.



Solution 6.2-3 **Hollow-box beam**



(1) Wood flanges

$b = 4$ in. $h = 12$ in. $h_1 = 8$ in.
$E_1 = 1{,}200{,}000$ psi
$(\sigma_1)_{allow} = 1700$ psi

(2) Plywood webs
$t = 1$ in. $h = 12$ in.
$E_2 = 1{,}600{,}000$ psi
$(\sigma_2)_{allow} = 2000$ psi

$$I_1 = \frac{b}{12}(h^3 - h_1^3) = 405.3 \text{ in.}^4$$

$$I_2 = 2\left(\frac{1}{12}\right)(th^3) = 288 \text{ in.}^4$$

$$E_1I_1 + E_2I_2 = 947.2 \times 10^6 \text{ lb-in.}^2$$

Maximum moment based upon the wood (1)

From Eq. (6-6a):

$$M_{max} = (\sigma_1)_{allow}\left[\frac{E_1I_1 + E_2I_2}{(h/2)E_1}\right] = 224 \text{ k-in.}$$

Maximum moment based upon the plywood (2)

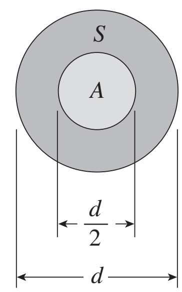
From Eq. (6-6b):

$$M_{max} = (\sigma_2)_{allow}\left[\frac{E_1I_1 + E_2I_2}{(h/2)E_2}\right] = 197 \text{ k-in.}$$

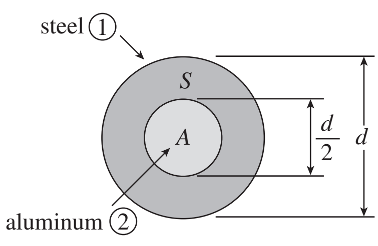
Plywood governs $M_{max} = 197$ k-in. ⟵

Problem 6.2-4 A round steel tube of outside diameter d and an aluminum core of diameter $d/2$ are bonded to form a composite beam as shown in the figure.

Derive a formula for the allowable bending moment M that can be carried by the beam based upon an allowable stress σ_s in the steel. (Assume that the moduli of elasticity for the steel and aluminum are E_s and E_a, respectively.)



Solution 6.2-4 **Steel tube with aluminum core**



Tube (1): d = outer diameter $\frac{d}{2}$ = inner diameter

E_s = modulus of elasticity
σ_s = allowable stress

$$I_1 = \frac{\pi}{64}[d^4 - (d/2)^4] = \frac{15\pi d^4}{1024}$$

Core (2): $d/2$ = diameter
E_a = modulus of elasticity

$$I_2 = \frac{\pi}{64}(d/2)^4 = \frac{\pi d^4}{1024}$$

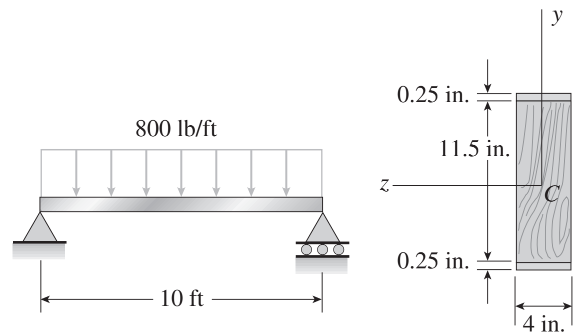
$$E_1I_1 + E_2I_2 = E_sI_1 + E_aI_2 = \frac{\pi d^4}{1024}(15E_s + E_a)$$

From Eq. (6-6a): $M_{allow} = \sigma_s\left[\frac{E_1I_1 + E_2I_2}{(d/2)E_s}\right]$

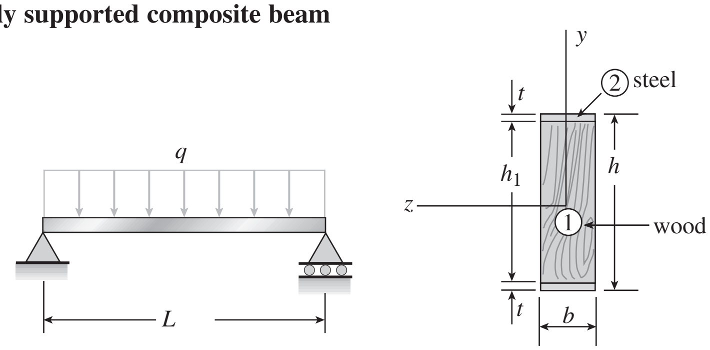
$$M_{allow} = \frac{\pi d^3 \sigma_3}{512}\left(15 + \frac{E_a}{E_s}\right) \longleftarrow$$

Problem 6.2-5 A simple beam on a 10 ft span supports a uniform load of intensity 800 lb/ft (see figure). The beam consists of a wood member (4 in. × 11.5 in. in cross section) that is reinforced by 0.25 in. thick steel plates on top and bottom. The moduli of elasticity for the steel and wood are $E_s = 30 \times 10^6$ psi and $E_w = 1.5 \times 10^6$ psi, respectively.

Calculate the maximum bending stresses σ_s in the steel plates and σ_w in the wood member due to the uniform load.



Solution 6.2-5 **Simply supported composite beam**



$L = 10$ ft $\quad q = 800$ lb/ft

$$M_{max} = \frac{qL^2}{8} = 10{,}000 \text{ lb-ft} = 120{,}000 \text{ lb-in.}$$

WOOD BEAM WITH STEEL PLATES

Wood (1): $b = 4$ in. $\quad h_1 = 11.5$ in.

$E_w = 1.5 \times 10^6$ psi

$$I_1 = \frac{bh_1^3}{12} = 506.96 \text{ in.}^4$$

Plate (2): $b = 4$ in. $\quad t = 0.25$ in. $\quad h = 12$ in.

$E_s = 30 \times 10^6$ psi

$$I_2 = \frac{b}{12}[h^3 - h_1^3] = 69.042 \text{ in.}^4$$

$$E_1I_1 + E_2I_2 = E_wI_1 + E_sI_2 = 2{,}832{,}000{,}000 \text{ lb-in.}^2$$

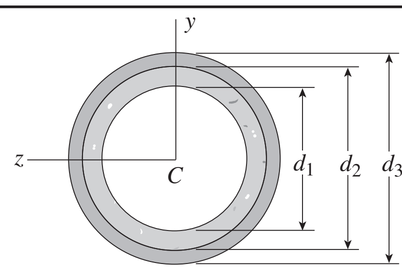
From Eqs. (6-6a and b):

$$\sigma_w = \frac{M_{max}(h_1/2)E_w}{E_1I_1 + E_2I_2} = 365 \text{ psi} \quad \longleftarrow$$

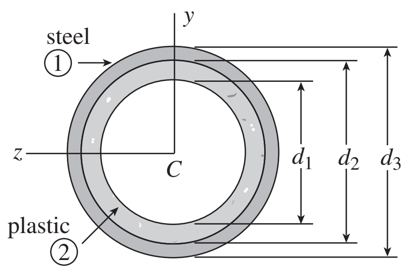
$$\sigma_s = \frac{M_{max}(h_1/2)E_s}{E_1I_1 + E_2I_2} = 7630 \text{ psi} \quad \longleftarrow$$

Problem 6.2-6 A plastic-lined steel pipe has the cross-sectional shape shown in the figure. The steel pipe has outer diameter $d_3 = 100$ mm and inner diameter $d_2 = 94$ mm. The plastic liner has inner diameter $d_1 = 82$ mm. The modulus of elasticity of the steel is 75 times the modulus of the plastic.

Determine the allowable bending moment M_{allow} if the allowable stress in the steel is 35 MPa and in the plastic is 600 kPa.



Solution 6.2-6 **Steel pipe with plastic liner**



(1) Pipe: $d_s = 100$ mm $\quad d_2 = 94$ mm
$E_s = E_1 =$ modulus of elasticity
$(\sigma_1)_{\text{allow}} = 35$ MPa

(2) Liner: $d_2 = 94$ mm $\quad d_1 = 32$ mm
$E_p = E_2 =$ modulus of elasticity
$(\sigma_2)_{\text{allow}} = 600$ kPa
$E_1 = 75E_2 \quad E_1/E_2 = 75$

$$I_1 = \frac{\pi}{64}(d_3^4 - d_2^4) = 1.076 \times 10^{-6}\ \text{m}^4$$

$$I_2 = \frac{\pi}{64}(d_2^4 - d_1^4) = 1.613 \times 10^{-6}\ \text{m}^4$$

Maximum moment based upon the steel (1)

From Eq. (6-6a):

$$M_{\max} = (\sigma_1)_{\text{allow}}\left[\frac{E_1I_1 + E_2I_2}{(d_3/2)E_1}\right]$$

$$= (\sigma_1)_{\text{allow}}\frac{(E_1/E_2)I_1 + I_2}{(d_s/2)(E_1/E_2)} = 768\ \text{N} \cdot \text{m}$$

Maximum moment based upon the plastic (2)

From Eq. (6-6b):

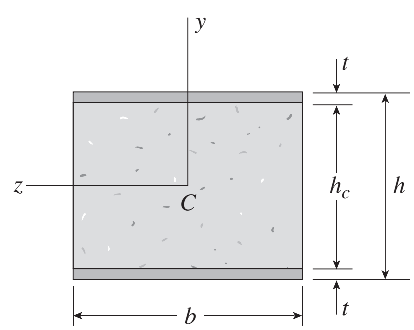
$$M_{\max} = (\sigma_2)_{\text{allow}}\left[\frac{E_1I_1 + E_2I_2}{(d_2/2)E_2}\right]$$

$$= (\sigma_2)_{\text{allow}}\left[\frac{(E_1/E_2)I_1 + I_2}{(d_2/2)}\right] = 1051\ \text{N} \cdot \text{m}$$

Steel governs $\quad M_{\text{allow}} = 768\ \text{N} \cdot \text{m}$ ←

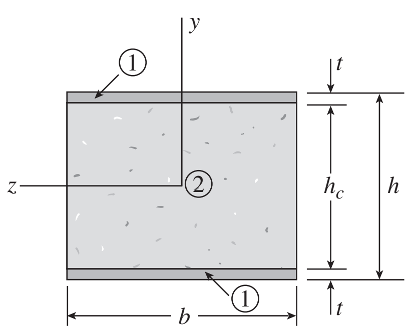
Problem 6.2-7 The cross section of a sandwich beam consisting of aluminum alloy faces and a foam core is shown in the figure. The width b of the beam is 8.0 in., the thickness t of the faces is 0.25 in., and the height h_c of the core is 5.5 in. (total height $h = 6.0$ in.). The moduli of elasticity are 10.5×10^6 psi for the aluminum faces and 12,000 psi for the foam core. A bending moment $M = 40$ k-in. acts about the z axis.

Determine the maximum stresses in the faces and the core using (a) the general theory for composite beams, and (b) the approximate theory for sandwich beams.



Probs. 6.2-7 and 6.2-8

Solution 6.2-7 **Sandwich beam**



(1) Aluminum faces: $b = 8.0$ in. $\quad t = 0.25$ in.
$h = 6.0$ in.
$E_1 = 10.5 \times 10^6$ psi

$$I_1 = \frac{b}{12}(h^3 - h_c^3) = 33.08\ \text{in.}^4$$

(2) Foam core: $b = 8.0$ in. $\quad h_c = 5.5$ in.
$E_2 = 12{,}000$ psi

$$I_2 = \frac{bh_c^3}{12} = 110.92\ \text{in.}^4$$

$M = 40$ k-in. $\quad E_1I_1 + E_2I_2 = 348.7 \times 10^6$ lb-in.^{2}

(a) General theory (Eqs. 6-6a and b)

$$\sigma_{\text{face}} = \sigma_1 = \frac{M(h/2)E_1}{E_1I_1 + E_2I_2} = 3610\ \text{psi} \quad \leftarrow$$

$$\sigma_{\text{core}} = \sigma_2 = \frac{M(h_c/2)E_2}{E_1I_1 + E_2I_2} = 4\ \text{psi} \quad \leftarrow$$

(b) Approximate theory (Eqs. 6-8 and 6-9)

$$I_1 = \frac{b}{12}(h^3 - h_c^3) = 33.08\ \text{in.}^4$$

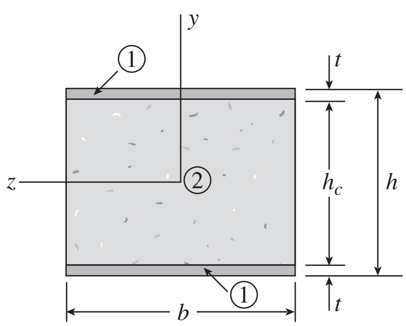
$$\sigma_{\text{face}} = \frac{Mb}{2I_1} = 3630\ \text{psi} \quad \leftarrow$$

$$\sigma_{\text{core}} = 0 \quad \leftarrow$$

Problem 6.2-8 The cross section of a sandwich beam consisting of fiberglass faces and a lightweight plastic core is shown in the figure. The width b of the beam is 50 mm, the thickness t of the faces is 4 mm, and the height h_c of the core is 92 mm (total height $h = 100$ mm). The moduli of elasticity are 75 GPa for the fiberglass and 1.2 GPa for the plastic. A bending moment $M = 275$ N · m acts about the z axis.

Determine the maximum stresses in the faces and the core using (a) the general theory for composite beams, and (b) the approximate theory for sandwich beams.

Solution 6.2-8 Sandwich beam



(1) Fiber glass faces: $b = 50$ mm $t = 4$ mm
$h = 100$ mm
$E_1 = 75$ GPa

$$I_1 = \frac{b}{12}(h^3 - h_c^3) = 0.9221 \times 10^{-6}\ \text{m}^4$$

(2) Plastic core: $b = 50$ mm $h_c = 92$ mm
$E_2 = 1.2$ GPa

$$I_2 = \frac{bh_c^3}{12} = 3.245 \times 10^{-6}\ \text{m}^4$$

$M = 275$ N · m $\quad E_1I_1 + E_2I_2 = 73{,}050$ N · m²

(a) General theory (Eqs. 6-6a and b)

$$\sigma_{\text{face}} = \sigma_1 = \frac{M(h/2)E_1}{E_1I_1 + E_2I_2} = 14.1\ \text{MPa} \longleftarrow$$

$$\sigma_{\text{core}} = \sigma_2 = \frac{M(h_c/2)E_2}{E_1I_1 + E_2I_2} = 0.21\ \text{MPa} \longleftarrow$$

(b) Approximate theory (Eqs. 6-8 and 6-9)

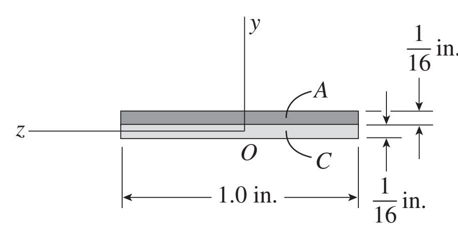
$$I_1 = \frac{b}{12}(h^3 - h_c^3) = 0.9221 \times 10^{6}\ \text{m}^4$$

$$\sigma_{\text{face}} = \frac{Mh}{2I_1} = 14.9\ \text{MPa} \longleftarrow$$

$$\sigma_{\text{core}} = 0 \longleftarrow$$

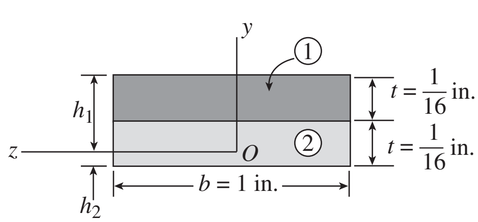
Problem 6.2-9 A bimetallic beam used in a temperature-control switch consists of strips of aluminum and copper bonded together as shown in the figure, which is a cross-sectional view. The width of the beam is 1.0 in., and each strip has a thickness of 1/16 in.

Under the action of a bending moment $M = 12$ lb-in. acting about the z axis, what are the maximum stresses σ_a and σ_c in the aluminum and copper, respectively? (Assume $E_a = 10.5 \times 10^6$ psi and $E_c = 16.8 \times 10^6$ psi.)



Solution 6.2-9 Bimetallic beam

Cross section



(1) Aluminum $E_1 = E_a = 10.5 \times 10^6$ psi
(2) Copper $E_2 = E_c = 16.8 \times 10^6$ psi
$M = 12$ lb-in.

Neutral axis (Eq. 6-3)

$$\int_1 Y\,dA = \bar{Y}_1A_1 = (h_1 - t/2)(bt)$$

$$= (h_1 - 1/32)(1)(1/16)\ \text{in.}^3$$

$$\int_2 Y\,dA = \bar{Y}_2 A_2 = (h_1 - t - t/2)(bt)$$

$$= (h_1 - 3/32)(1)(1/16) \text{ in.}^3$$

Eq. (6-3): $E_1 \int_1 Y dA + E_2 \int_2 Y dA = 0$

$(10.5 \times 10^6)(h_1 - 1/32)(1/16)$

$+ (16.8 \times 10^6)(h_1 - 3/32)(1/16) = 0$

Solve for h_1: $h_1 = 0.06971$ in.

$h_2 = 2(1/16 \text{ in.}) - h_1 = 0.05529$ in.

MOMENTS OF INERTIA (FROM PARALLEL-AXIS THEOREM)

$$I_1 = \frac{bt^3}{12} + bt(h_1 - t/2)^2 = 0.0001128 \text{ in.}^4$$

$$I_2 = \frac{bt^3}{12} + bt(h_2 - t/2)^2 = 0.00005647 \text{ in.}^4$$

$$E_1 I_1 + E_2 I_2 = 2133 \text{ lb-in.}^2$$

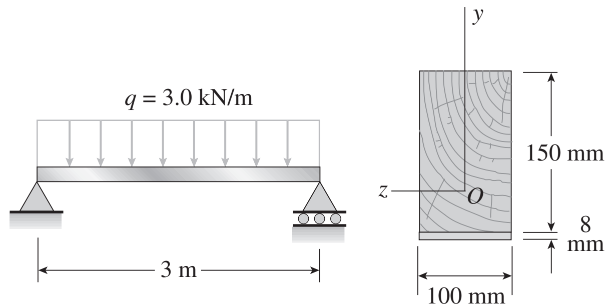
MAXIMUM STRESSES (EQS. 6-6a AND b)

$$\sigma_a = \sigma_1 = \frac{M h_1 E_1}{E_1 I_1 + E_2 I_2} = 4120 \text{ psi} \quad \longleftarrow$$

$$\sigma_c = \sigma_2 = \frac{M h_2 E_2}{E_1 I_1 + E_2 I_2} = 5230 \text{ psi} \quad \longleftarrow$$

Problem 6.2-10 A simply supported composite beam 3 m long carries a uniformly distributed load of intensity $q = 3.0$ kN/m (see figure). The beam is constructed of a wood member, 100 mm wide by 150 mm deep, reinforced on its lower side by a steel plate 8 mm thick and 100 mm wide.

Find the maximum bending stresses σ_w and σ_s in the wood and steel, respectively, due to the uniform load if the moduli of elasticity are $E_w = 10$ GPa for the wood and $E_s = 210$ GPa for the steel.

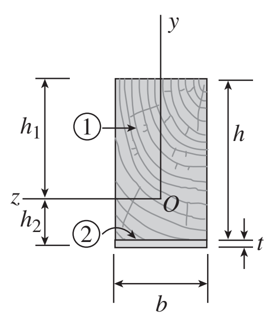


Solution 6.2-10 Simply supported composite beam

BEAM: $L = 3$ m $\quad q = 3.0$ kN/m

$$M_{max} = \frac{qL^2}{8} = 3375 \text{ N} \cdot \text{m}$$

CROSS SECTION



$b = 100$ mm $\quad h = 150$ mm $\quad t = 8$ mm

(1) Wood: $E_1 = E_w = 10$ GPa

(2) Steel: $E_2 = E_s = 210$ GPa

NEUTRAL AXIS

$$\int_1 Y\,dA = \bar{Y}_1 A_1 = (h_1 - h/2)(bh)$$

$$= (h_1 - 75)(100)(150) \text{ mm}^3$$

$$\int_2 y\,dA = \bar{Y}_2 A_2 = -(h + t/2 - h_1)(bt)$$

$$= -(154 - h_1)(100)(18) \text{ mm}^3$$

Eq. (6-3): $E_1 \int_1 Y dA + E_2 \int_2 Y dA = 0$

$(10 \text{ GPa})(h_1 - 75)(100)(150)(10^{-9})$

$+ (210 \text{ GPa})(h_1 - 154)(100)(8)(10^{-9}) = 0$

Solve for h_1: $h_1 = 116.74$ mm

$h_2 = h + t - h_1 = 41.26$ mm

MOMENTS OF INERTIA (FROM PARALLEL-AXIS THEOREM)

$$I_1 = \frac{bh^3}{12} + bh(h_1 - h/2)^2 = 54.26 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{bt^2}{12} + bt(h_2 - t/2)^2 = 1.115 \times 10^6 \text{ mm}^4$$

$$E_1 I_1 + E_2 I_2 = 776{,}750 \text{ N} \cdot \text{m}^2$$

MAXIMUM STRESSES (EQS. 6-6a AND b)

$$\sigma_w = \sigma_1 = \frac{M h_1 E_1}{E_1 I_1 + E_2 I_2}$$

$$= 5.1 \text{ MPa (Compression)} \quad \longleftarrow$$

$$\sigma_s = \sigma_2 = \frac{M h_2 E_2}{E_1 I_1 + E_2 I_2}$$

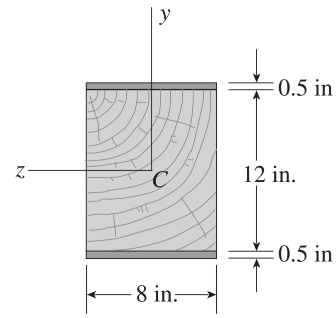
$$= 37.6 \text{ MPa (Tension)} \quad \longleftarrow$$

Transformed-Section Method

When solving the problems for Section 6.3, assume that the component parts of the beams are securely bonded by adhesives or connected by fasteners. Also, be sure to use the transformed-section method in the solutions.

Problem 6.3-1 A wood beam 8 in. wide and 12 in. deep is reinforced on top and bottom by 0.5 in. thick steel plates (see figure).

Find the allowable bending moment M_{max} about the z axis if the allowable stress in the wood is 1,000 psi and in the steel is 16,000 psi. (Assume that the ratio of the moduli of elasticity of steel and wood is 20.)

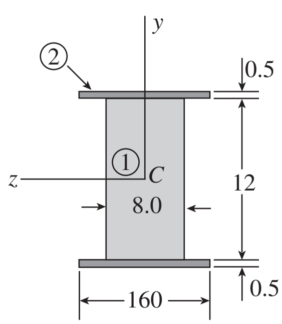


Solution 6.3-1 Wood beam with steel plates

(1) Wood beam $b = 8$ in. $h_1 = 12$ in.
$(\sigma_1)_{allow} = 1000$ psi

(2) Steel plates $b = 8$ in. $h_2 = 13$ in.
$t = 0.5$ in.
$(\sigma_2)_{allow} = 16{,}000$ psi

TRANSFORMED SECTION (WOOD)



Wood beam is not changed.

$$n = \frac{E_s}{E_w} = 20$$

Width of steel plates
$= nb = (20)(8 \text{ in.}) = 160$ in.

All dimensions in inches.

$$I_T = 1/12(160)(13)^3 - 1/12(160 - 8)(12)^3 = 7405 \text{ in.}^4$$

MAXIMUM MOMENT BASED UPON THE WOOD (1) (EQ. 6-15)

$$\sigma_1 = \frac{M(h_1/2)}{I_T} \qquad M_1 = \frac{(\sigma_1)_{allow} I_T}{h_1/2} = 1230 \text{ k-in.}$$

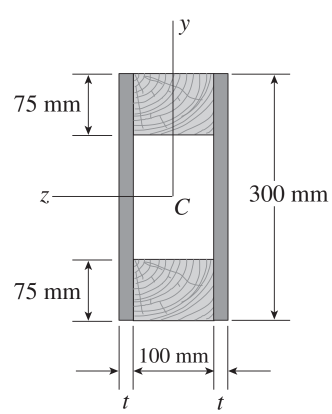
MAXIMUM MOMENT BASED UPON THE STEEL (2) (EQ. 6-17)

$$\sigma_2 = \frac{M(h_2/2)n}{I_T} \qquad M_2 = \frac{(\sigma_2)_{allow} I_T}{h_1/2\,n} = 911 \text{ k-in.}$$

STEEL GOVERNS $M_{max} = 911$ k-in. ←

Problem 6.3-2 A simple beam of span length 3.2 m carries a uniform load of intensity 48 kN/m. The cross section of the beam is a hollow box with wood flanges and steel side plates, as shown in the figure. The wood flanges are 75 mm by 100 mm in cross section, and the steel plates are 300 mm deep.

What is the required thickness t of the steel plates if the allowable stresses are 120 MPa for the steel and 6.5 MPa for the wood? (Assume that the moduli of elasticity for the steel and wood are 210 GPa and 10 GPa, respectively, and disregard the weight of the beam.)



Solution 6.3-2 **Box beam**

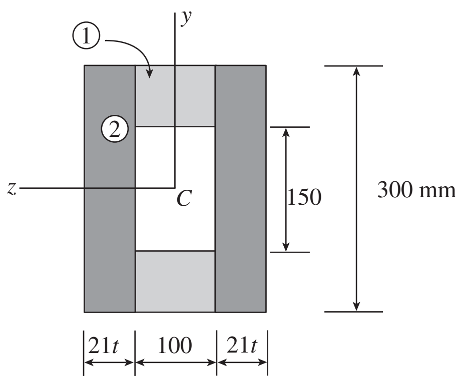
$M_{\text{MAX}} = \frac{qL^2}{8} = 61.44 \text{ kN} \cdot \text{m}$

SIMPLE BEAM: $L = 3.2$ m $\quad q = 4.8$ kN/m

(1) Wood flanges: $b = 100$ mm $\quad h = 300$ mm
$h_1 = 150$ mm
$(\sigma_1)_{\text{allow}} = 6.5$ MPa
$E_w = 10$ GPa

(2) Steel plates: t = thickness $\quad h = 300$ mm
$(\sigma_2)_{\text{allow}} = 120$ MPa
$E_s = 210$ GPa

TRANSFORMED SECTION (WOOD)



Wood flanges are not changed

$n = \frac{E_s}{E_w} = 21$

Width of steel plates
$= nt = 21t$

All dimensions in millimeters.

$$I_T = \frac{1}{12}(100 + 42t)(300)^3 - \frac{1}{12}(100)(150)^3$$
$$= 196.9 \times 10^6 \text{ mm}^4 + 94.5t \times 10^6 \text{ mm}^4$$

REQUIRED THICKNESS BASED UPON THE WOOD (1) (EQ. 6-15)

$$\sigma_1 = \frac{M(h/2)}{I_T} \qquad (I_r)_1 = \frac{M_{\max}(h/2)}{(\sigma_1)_{\text{allow}}}$$
$$= 1.418 \times 10^9 \text{ mm}^4$$

Equate I_T and $(I_T)_1$ and solve for t: $t_1 = 12.92$ mm

REQUIRED THICKNESS BASED UPON THE STEEL (2) (EQ. 6-17)

$$\sigma_2 = \frac{M(h/2)n}{I_T} \qquad (I_T)_2 = \frac{M_{\max}(h/2)n}{(\sigma_2)_{\text{allow}}}$$
$$= 1.612 \times 10^9 \text{ mm}^4$$

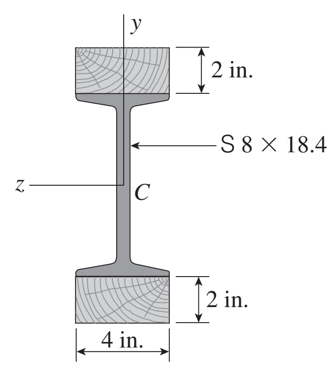
Equate I_r and $(I_r)_2$ and solve for t: $t_2 = 14.97$ mm

STEEL GOVERNS. $\quad t_{\min} = 15.0$ mm ←

Problem 6.3-3 A simple beam that is 15 ft long supports a uniform load of intensity q. The beam is constructed of an S 8 × 18.4 section (I-beam section) reinforced with wood beams that are securely fastened to the flanges (see the cross section shown in the figure). The wood beams are 2 in. deep and 4 in. wide. The modulus of elasticity of the steel is 20 times that of the wood.

If the allowable stresses in the steel and wood are 12,000 psi and 900 psi, respectively, what is the allowable load q_{allow}?

(*Note:* Disregard the weight of the beam, and see Table E-2 of Appendix E for the dimensions and properties of the steel beam.)



Solution 6.3-3 Reinforced I-beam

(1) Steel beam: S 8 × 18.4 $I_1 = 57.6 \text{ in.}^4$

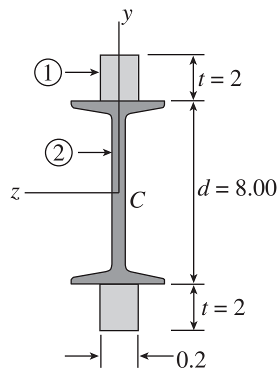
$d = 8.00$ in.

$(\sigma_1)_{\text{allow}} = 12{,}000$ psi

(2) Wood beams: $b = 4$ in. $t = 2$ in.

$(\sigma_2)_{\text{allow}} = 900$ psi

TRANSFORMED SECTION (STEEL)



Steel beam is not changed.

$$n = \frac{E_w}{E_s} = \frac{1}{20}$$

Width of wood beams

$= nb = (1/20)(4) = 0.2$ in.

All dimensions in inches.

$$I_T = 57.6 + \frac{1}{12}(0.2)(12)^3 - \frac{1}{12}(0.2)(8)^3$$
$$= 77.87 \text{ in.}^4$$

MAXIMUM MOMENT BASED UPON THE STEEL (1) (EQ. 6-15)

$$\sigma_1 = \frac{M(d/2)}{I_T} \qquad M_1 = \frac{(\sigma_1)_{\text{allow}} I_T}{d/2}$$
$$= 233{,}610 \text{ lb-in.}$$

MAXIMUM MOMENT BASED UPON THE WOOD (2) (EQ. 6-17)

$$\sigma_2 = \frac{M(d/2 + t)n}{I_T} \qquad M_2 = \frac{(\sigma_2)_{\text{allow}} I_T}{(d/2 + t)n}$$
$$= 233{,}610 \text{ lb-in.}$$

By coincidence, $M_1 = M_2$ (exactly)

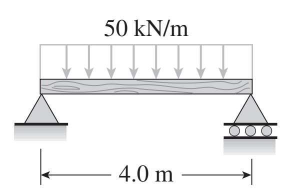
$M_{\max} = 233{,}610$ lb-in.

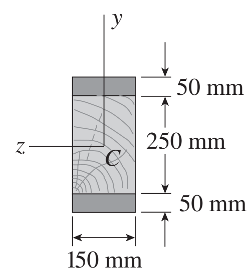
SIMPLE BEAM $L = 15$ ft

$$M_{\max} = \frac{qL^2}{8} \qquad q_{\text{allow}} = \frac{8M_{\max}}{L^2} = 692 \text{ lb/ft} \quad \longleftarrow$$

Problem 6.3-4 The composite beam shown in the figure is simply supported and carries a total uniform load of 50 kN/m on a span length of 4.0 m. The beam is built of a wood member having cross-sectional dimensions 150 mm × 250 mm and two steel plates of cross-sectional dimensions 50 mm × 150 mm.

Determine the maximum stresses σ_s and σ_w in the steel and wood, respectively, if the moduli of elasticity are $E_s = 209$ GPa and $E_w = 11$ GPa. (Disregard the weight of the beam.)





Solution 6.3-4 Composite beam

SIMPLE BEAM: $L = 4.0$ m $q = 50$ kN/m

$$M_{\max} = \frac{qL^2}{8} = 100 \text{ kN} \cdot \text{m}$$

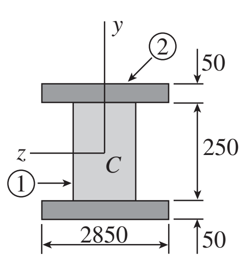
(1) Wood beam: $b = 150$ mm $h_1 = 250$ mm

$E_w = 11$ GPa

(2) Steel plates: $b = 150$ mm $t = 50$ mm

$h_2 = 350$ mm $E_s = 209$ GPa

TRANSFORMED SECTION (WOOD)



Wood beam is not changed.

$$n = \frac{E_s}{E_w} = \frac{209}{11} = 19$$

Width of steel plates
$= nb = (19)(150 \text{ mm}) = 2850 \text{ mm}$

All dimensions in millimeters.

$$I_T = \frac{1}{12}(2850)(350)^3 - \frac{1}{12}(2850 - 150)(250)^3$$
$$= 6.667 \times 10^8 \text{ mm}^4$$

MAXIMUM STRESS IN THE WOOD (1) (EQ. 6-15)

$$\sigma_w = \sigma_1 = \frac{M_{max}(h_1/2)}{I_T} = 1.9 \text{ MPa} \longleftarrow$$

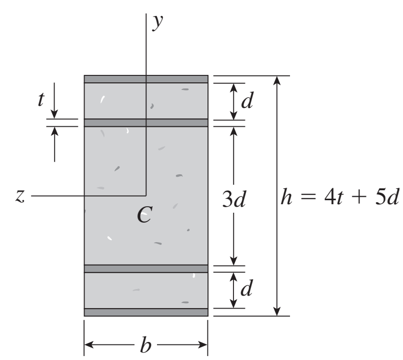
MAXIMUM STRESS IN THE STEEL (2) (EQ. 6-17)

$$\sigma_s = \sigma_2 = \frac{M_{max}(h_2/2)n}{I_T} = 49.9 \text{ MPa} \longleftarrow$$

Problem 6.3-5 The cross section of a beam made of thin strips of aluminum separated by a lightweight plastic is shown in the figure. The beam has width $b = 3.0$ in., the aluminum strips have thickness $t = 0.1$ in., and the plastic segments have heights $d = 1.2$ in. and $3d = 3.6$ in. The total height of the beam is $h = 6.4$ in.

The moduli of elasticity for the aluminum and plastic are $E_a = 11 \times 10^6$ psi and $E_p = 440 \times 10^3$ psi, respectively.

Determine the maximum stresses σ_a and σ_p in the aluminum and plastic, respectively, due to a bending moment of 6.0 k-in.



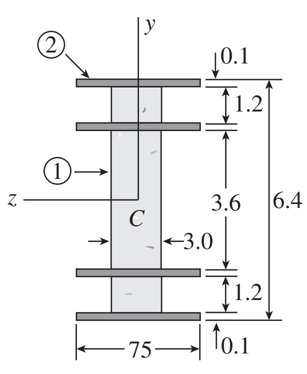
Probs. 6.3-5 and 6.3-6

Solution 6.3-5 Plastic beam with aluminum strips

(1) Plastic segments: $b = 3.0$ in. $d = 1.2$ in.
$3d = 3.6$ in.
$E_p = 440 \times 10^3$ psi

(2) Aluminum strips: $b = 3.0$ in. $t = 0.1$ in.
$E_a = 11 \times 10^6$ psi
$h = 4t + 5d = 6.4$ in.
$M = 6.0$ k-in.

TRANSFORMED SECTION (PLASTIC)



Plastic segments are not changed.

$$n = \frac{E_a}{E_p} = 25$$

Width of aluminum strips
$= nb = (25)(3.0 \text{ in.}) = 75 \text{ in.}$

All dimensions in inches.

$$\text{Plastic: } I_1 = 2\left[\frac{1}{12}(3.0)(1.2)^3 + (3.0)(1.2)(2.50)^2\right] + \frac{1}{12}(3.0)(3.6)^3 = 57.528 \text{ in.}^4$$

Aluminum:

$$I_2 = 2\left[\frac{1}{12}(75)(0.1)^3 + (75)(0.1)(3.15)^2 + \frac{1}{12}(75)(0.1)^3 + (75)(0.1)(1.85)^2\right]$$
$$= 200.2 \text{ in.}^4$$
$$I_T = I_1 + I_2 = 257.73 \text{ in.}^4$$

MAXIMUM STRESS IN THE PLASTIC (1) (Eq. 6-15)

$$\sigma_p = \sigma_1 = \frac{M(h/2 - t)}{I_T} = 72 \text{ psi} \longleftarrow$$

MAXIMUM STRESS IN THE ALUMINUM (2) (EQ. 6-17)

$$\sigma_a = \sigma_2 = \frac{M(h/2)n}{I_T} = 1860 \text{ psi} \longleftarrow$$

Problem 6.3-6 Consider the preceding problem if the beam has width $b = 75$ mm, the aluminum strips have thickness $t = 3$ mm, the plastic segments have heights $d = 40$ mm and $3d = 120$ mm, and the total height of the beam is $h = 212$ mm. Also, the moduli of elasticity are $E_a = 75$ GPa and $E_p = 3$ GPa, respectively.

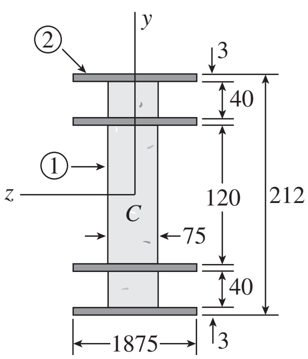
Determine the maximum stresses σ_a and σ_p in the aluminum and plastic, respectively, due to a bending moment of 1.0 kN · m.

Solution 6.3-6 Plastic beam with aluminum strips

(1) Plastic segments: $b = 75$ mm $d = 40$ mm
$3d = 120$ mm $E_p = 3$ GPa

(2) Aluminum strips: $b = 75$ mm $t = 3$ mm
$E_a = 75$ GPa
$h = 4t + 5d = 212$ mm
$M = 1.0$ kN · m

TRANSFORMED SECTION (PLASTIC)



Plastic segments are not changed.

$$n = \frac{E_a}{E_p} = 25$$

Width of aluminum strips
$= nb = (25)(75 \text{ mm}) = 1875$ mm

All dimensions in millimeters.

$$\text{Plastic: } I_1 = 2\left[\frac{1}{12}(75)(40)^3 + (75)(40)(83)^2\right] + \frac{1}{12}(75)(120)^3 = 52.934 \times 10^6 \text{ mm}^4$$

Aluminum:

$$I_2 = 2\left[\frac{1}{12}(1875)(3)^3 + (1875)(3)(104.5)^2 + \frac{1}{12}(1875)(3)^3 + (1875)(3)(61.5)^2\right] = 165.420 \times 10^6 \text{ mm}^4$$

$$I_T = I_1 + I_2 = 218.35 \times 10^6 \text{ mm}^4$$

MAXIMUM STRESS IN THE PLASTIC (1) (EQ. 6-15)

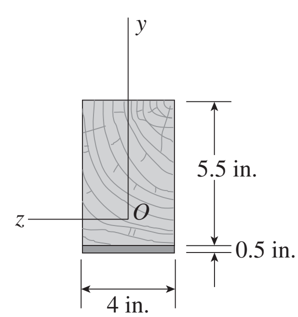
$$\sigma_p = \sigma_1 = \frac{M(h/2 - t)}{I_T} = 0.47 \text{ MPa} \longleftarrow$$

MAXIMUM STRESS IN THE ALUMINUM (2) (EQ. 6-17)

$$\sigma_a = \sigma_2 = \frac{M(h/2)n}{I_T} = 12.14 \text{ MPa} \longleftarrow$$

Problem 6.3-7 A composite beam constructed of a wood beam reinforced by a steel plate has the cross-sectional dimensions shown in the figure. The beam is simply supported with a span length of 6.0 ft and supports a uniformly distributed load of intensity $q = 800$ lb/ft.

Calculate the maximum bending stresses σ_s and σ_w in the steel and wood, respectively, due to the uniform load if $E_s/E_w = 20$.



Solution 6.3-7 **Composite beam**

SIMPLE BEAM: $L = 6.0$ ft $q = 800$ lb/ft

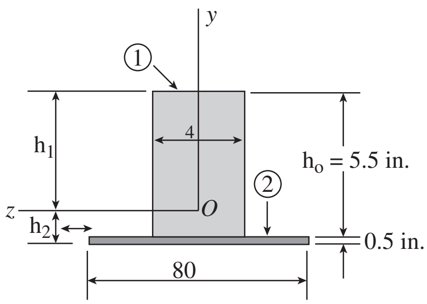
$M_{max} = qL^2/8 = 43{,}200$ lb-in.

(1) Wood beam: $b = 4$ in. $h_0 = 5.5$ in.

(2) Steel plate: $b = 4$ in. $t = 0.5$ in.

$h = h_0 + t = 6.0$ in. $\dfrac{E_s}{E_w} = 20$

TRANSFORMED SECTION (WOOD)



Wood beam is not changed. $n = 20$

Width of steel plate $= nb = (20)(4 \text{ in.}) = 80$ in.

All dimensions in inches.

Use the base of the cross section as a reference line.

$$h_2 = \frac{\sum y_c A_c}{\sum A_c} = \frac{(0.25)(80)(0.5) + (3.25)(4)(5.5)}{(80)(0.5) + (4)(5.5)}$$

$$= 1.3145 \text{ in.}$$

$$h_1 = h - h_2 = 4.6855 \text{ in.}$$

$$I_T = \frac{1}{12}(4)(5.5)^3 + (4)(5.5)(h_1 - 2.75)^2$$

$$+ \frac{1}{12}(80)(0.5)^3 + (80)(0.5)(h_2 - 0.25)^2$$

$$= 184.03 \text{ in.}^4$$

MAXIMUM STRESS IN THE WOOD (1) (EQ. 6-15)

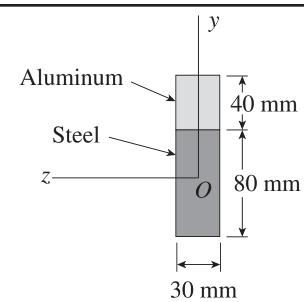
$$\sigma_w = \sigma_1 = \frac{M_{max} h_1}{I_T} = 1100 \text{ psi (Compression)} \quad \longleftarrow$$

MAXIMUM STRESS IN THE STEEL (2) (EQ. 6-17)

$$\sigma_s = \sigma_2 = \frac{M_{max} h_2 n}{I_T} = 6170 \text{ psi (Tension)} \quad \longleftarrow$$

Problem 6.3-8 The cross section of a composite beam made of aluminum and steel is shown in the figure. The moduli of elasticity are $E_a = 75$ GPa and $E_s = 200$ GPa.

Under the action of a bending moment that produces a maximum stress of 50 MPa in the aluminum, what is the maximum stress σ_s in the steel?

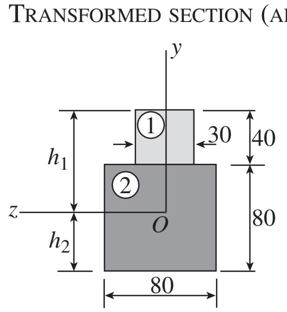


Solution 6.3-8 **Composite beam of aluminum and steel**

(1) Aluminum: $b = 30$ mm $h_a = 40$ mm
$E_a = 75$ GPa $\sigma_a = 50$ MPa

(2) Steel: $b = 30$ mm $h_s = 80$ mm
$E_s = 200$ GPa $\sigma_s = ?$

TRANSFORMED SECTION (ALUMINUM)



Aluminum part is not changed.

$$n = \frac{E_s}{E_a} = \frac{200}{75} = 2.667$$

Width of steel part

$= nb = (2.667)(30 \text{ mm}) = 80$ mm

All dimensions in millimeters.

Use the base of the cross section as a reference line.

$$h_2 = \frac{\sum y_i A_i}{\sum A_i} = \frac{(40)(80)(80) + (100)(30)(40)}{(80)(80) + (30)(40)}$$

$$= 49.474 \text{ mm}$$

$$h_1 = 120 - h_2 = 70.526 \text{ mm}$$

MAXIMUM STRESS IN THE ALUMINUM (1) (EQ. 6-15)

$$\sigma_a = \sigma_1 = \frac{M h_1}{I_T}$$

MAXIMUM STRESS IN THE STEEL (2) (EQ. 6-17)

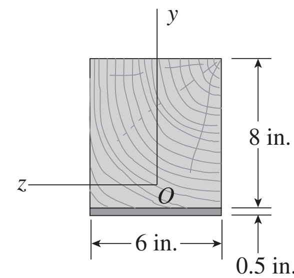
$$\sigma_s = \sigma_2 = \frac{M h_2 n}{I_T}$$

$$\frac{\sigma_s}{\sigma_a} = \frac{h_2 n}{h_1} = \frac{(49.474)(2.667)}{70.526} = 1.8707$$

$$\sigma_s = 1.8707 \text{ (50 MPa)} = 93.5 \text{ MPa} \quad \longleftarrow$$

Problem 6.3-9 A composite beam is constructed of a wood beam 6 in. wide and 8 in. deep reinforced on the lower side by a 0.5 in. by 6 in. steel plate (see figure). The modulus of elasticity for the wood is $E_w = 1.2 \times 10^6$ psi and for the steel is $E_s = 30 \times 10^6$ psi.

Find the allowable bending moment M_{allow} for the beam if the allowable stress in the wood is $\sigma_w = 1200$ psi and in the steel is $\sigma_s = 10{,}000$ psi.



Solution 6.3-9 Composite beam of wood and steel

(1) Wood beam: $b = 6$ in. $h_w = 8$ in.

$E_w = 1.2 \times 10^6$ psi

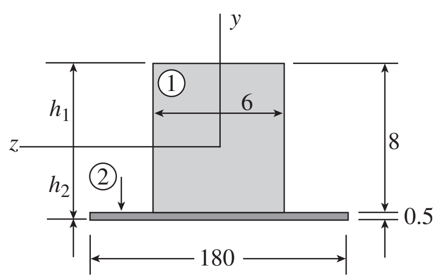
$(\sigma_w)_{allow} = 1200$ psi

(2) Steel plate: $b = 6$ in. $t = 0.5$ in.

$E_s = 30 \times 10^6$ psi

$(\sigma_s)_{allow} = 10{,}000$ psi

Transformed section (wood)



Wood beam is not changed.

$$n = \frac{E_s}{E_w} = \frac{30}{1.2} = 25$$

Width of steel plate $= nb = (25)(6 \text{ in.}) = 150$ in.

All dimensions in inches.

Use the base of the cross section as a reference line.

$$h_2 = \frac{\sum y_i A_i}{\sum A_i} = \frac{(0.25)(150)(0.5) + (4.5)(6)(8)}{(150)(0.5) + (6)(8)}$$

$= 1.9085$ in.

$h_1 = 8.5 - h_2 = 6.5915$ in.

$$I_T = \frac{1}{12}(6)(8)^3 + (6)(8)(h_1 - 4)^2 + \frac{1}{12}(150)(0.5)^3 + (150)(0.5)(h_2 - 0.25)^2 = 786.22 \text{ in.}^4$$

Maximum moment based upon the wood (1) (Eq. 6-15)

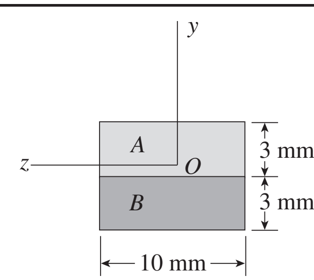
$$\sigma_w = \sigma_1 = \frac{Mh_1}{I_T} \qquad M_1 = \frac{(\sigma_w)_{allow} I_T}{h_1} = 143 \text{ k-in.}$$

Maximum moment based upon the steel (2) (Eq. 6-17)

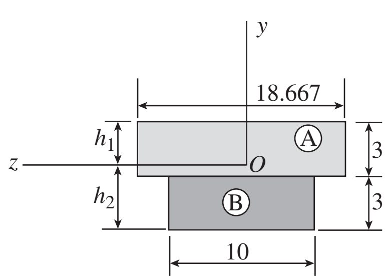
$$\sigma_s = \sigma_2 = \frac{Mh_2 n}{I_T} \qquad M_2 = \frac{(\sigma_s)_{allow} I_T}{h_2 n} = 165 \text{ k-in.}$$

Wood governs. $M_{allow} = 143$ k-in. ←

Problem 6.3-10 The cross section of a bimetallic strip is shown in the figure. Assuming that the moduli of elasticity for metals A and B are $E_A = 168$ GPa and $E_B = 90$ GPa, respectively, determine the smaller of the two section moduli for the beam. (Recall that section modulus is equal to bending moment divided by maximum bending stress.) In which material does the maximum stress occur?



Solution 6.3-10 Bimetallic strip



Metal A: $b = 10$ mm $h_A = 3$ mm

$E_A = 168$ GPa

Metal B: $b = 10$ mm

$h_B = 3$ mm $E_B = 90$ GPa

TRANSFORMED SECTION (METAL B)

Metal B does not change.

$$n = \frac{E_A}{E_B} = \frac{168}{90} = 1.8667$$

Width of metal A

$= nb = (1.8667)(10 \text{ mm}) = 18.667 \text{ mm}$

All dimensions in millimeters.

Use the base of the cross section as a reference line.

$$h_2 = \frac{\sum y_i A_i}{\sum A_i} = \frac{(1.5)(10)(13) + (4.5)(18.667)(3)}{(10)(3) + (18.667)(3)}$$

$= 3.4535 \text{ mm}$

$h_1 = 6 - h_2 = 2.5465 \text{ mm}$

$$I_r = \frac{1}{12}(10)(3)^3 + (10)(3)(h_2 - 1.5)^2 + \frac{1}{12}(18.667)(3)^3 + (18.667)(3)(h_1 - 1.5)^2$$

$= 240.31 \text{ mm}^4$

MAXIMUM STRESS IN MATERIAL B (EQ. 6-15)

$$\sigma_B = \sigma_1 = \frac{Mh_2}{I_r} \qquad S_B = \frac{M}{\sigma_B} = \frac{I_T}{h_2} = 69.6 \text{ mm}^3$$

MAXIMUM STRESS IN MATERIAL A (EQ. 6-17)

$$\sigma_A = \sigma_2 = \frac{Mh_1 n}{I_T} \qquad S_A = \frac{M}{\sigma_A} = \frac{I_r}{h_1 n}$$

$= 50.6 \text{ mm}^3$

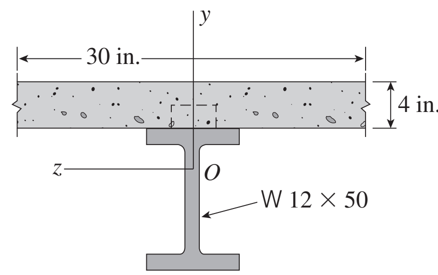
SMALLER SECTION MODULUS

$S_A = 50.6 \text{ mm}^3$ ←

∴ Maximum stress occurs in metal A. ←

Problem 6.3-11 A W 12 × 50 steel wide-flange beam and a segment of a 4-inch thick concrete slab (see figure) jointly resist a positive bending moment of 95 k-ft. The beam and slab are joined by shear connectors that are welded to the steel beam. (These connectors resist the horizontal shear at the contact surface.) The moduli of elasticity of the steel and the concrete are in the ratio 12 to 1.

Determine the maximum stresses σ_s and σ_c in the steel and concrete, respectively. (*Note:* See Table E-1 of Appendix E for the dimensions and properties of the steel beam.)



Solution 6.3-11 Steel beam and concrete slab

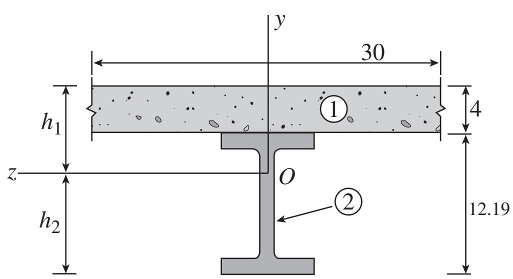
(1) Concrete: $b = 30$ in. $t = 4$ in.

(2) Wide-flange beam: W 12 × 50

$d = 12.19$ in. $I = 394 \text{ in.}^4$

$A = 14.7 \text{ in.}^2$ $M = 95 \text{ k-ft} = 1140 \text{ k-in.}$

TRANSFORMED SECTION (CONCRETE)



No change in dimensions of the concrete.

$$n = \frac{E_s}{E_c} = \frac{E_2}{E_1} = 12$$

Width at steel beam is increased by the factor n to transform to concrete.

All dimensions in inches.

Use the base of the cross section as a reference line.

$nI = 4728 \text{ in.}^4$ $nA = 176.4 \text{ in.}^2$

$$h_2 = \frac{\sum y_i A_i}{\sum A_i} = \frac{(12.19/2)(176.4) + (14.19)(30)(4)}{176.4 + (30)(4)}$$

$= 9.372$ in.

$h_1 = 16.19 - h_2 = 6.818$ in.

$$I_T = \frac{1}{12}(30)(4)^3 + (30)(4)(h_1 - 2)^2 + 4728 + (176.4)(h_2 - 12.19/2)^2 = 9568 \text{ in.}^4$$

MAXIMUM STRESS IN THE CONCRETE (1) (EQ. 6-15)

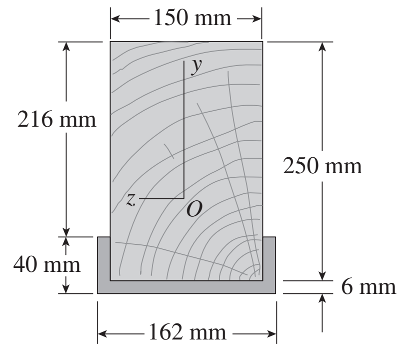
$$\sigma_c = \sigma_1 = \frac{Mh_1}{I_T} = 812 \text{ psi (Compression)}$$ ←

MAXIMUM STRESS IN THE STEEL (2) (EQ. 6-17)

$$\sigma_s = \sigma_2 = \frac{Mh_2 n}{I_T} = 13{,}400 \text{ psi (Tension)}$$ ←

Problem 6.3-12 A wood beam reinforced by an aluminum channel section is shown in the figure. The beam has a cross section of dimensions 150 mm by 250 mm, and the channel has a uniform thickness of 6 mm.

If the allowable stresses in the wood and aluminum are 8.0 MPa and 38 MPa, respectively, and if their moduli of elasticity are in the ratio 1 to 6, what is the maximum allowable bending moment for the beam?

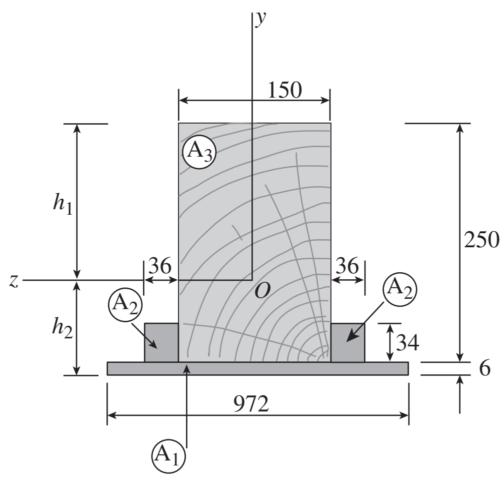


Solution 6.3-12 Wood beam and aluminum channel

(1) Wood beam: $b_w = 150$ mm $h_w = 250$ mm
$(\sigma_w)_{allow} = 8.0$ MPa

(2) Aluminum channel: $t = 6$ mm $b_a = 162$ mm
$h_a = 40$ mm
$(\sigma_a)_{allow} = 38$ MPa

TRANSFORMED SECTION (WOOD)



Wood beam is not changed.

$$n = \frac{E_a}{E_w} = 6$$

Width of aluminum channel is increased.

$nb_a = (6)(162\text{ mm}) = 972$ mm
$nt = (6)(6\text{ mm}) = 36$ mm

All dimensions in millimeters.

Use the base of the cross section as a reference line.

$$h_2 = \frac{\sum y_i A_i}{\sum A_i}$$

Area A_1: $y_1 = 3$ $A_1 = (972)(6) = 5832$
$y_1 A_1 = 17{,}496\text{ mm}^3$

Area A_2: $y_2 = 23$ $A_2 = (36)(34) = 1224$
$y_2 A_2 = 28{,}152\text{ mm}^3$

Area A_3: $y_3 = 131$ $A_3 = (150)(250) = 37{,}500$
$y_3 A_3 = 4{,}912{,}500\text{ mm}^3$

$$h_2 = \frac{y_1 A_1 + 2y_2 A_2 + y_3 A_3}{A_1 + 2A_2 + A_3} = \frac{4{,}986{,}300\text{ mm}^3}{45{,}780\text{ mm}^2}$$
$$= 108.92\text{ mm}$$

$h_1 = 256 - h_2 = 147.08$ mm

MOMENT OF INERTIA

Area A_1: $I_1 = \frac{1}{12}(972)(6)^3 + (972)(6)(h_2 - 3)^2$
$= 65{,}445{,}000\text{ mm}^4$

Area A_2: $I_2 = \frac{1}{12}(36)(34)^3$
$+ (36)(34)(h_2 - 6 - 17)^2$
$= 9{,}153{,}500\text{ mm}^4$

Area A_3: $I_3 = \frac{1}{12}(150)(250)^3$
$+ (150)(250)(h_1 - 125)^2$
$= 213{,}597{,}000\text{ mm}^4$

$I_T = I_1 + 2I_2 + I_3 = 297.35 \times 10^6\text{ mm}^4$

MAXIMUM MOMENT BASED UPON THE WOOD (1)
(EQ. 6-15)

$$\sigma_w = \sigma_1 = \frac{Mh_1}{I_T} \qquad M_1 = \frac{(\sigma_w)_{allow} I_T}{h_1} = 16.2\text{ kN}\cdot\text{m}$$

MAXIMUM MOMENT BASED UPON ALUMINUM (2)
(EQ. 6-17)

$$\sigma_a = \sigma_2 = \frac{Mh_2 n}{I_T} \qquad M_2 = \frac{(\sigma_a)_{allow} I_T}{h_2 n} = 17.3\text{ kN}\cdot\text{m}$$

WOOD GOVERNS. $M_{allow} = 16.2\text{ kN}\cdot\text{m}$ ⟵